![Instron logo]

Statistics and Charts

TrendTracker Module in Bluehill® Central

The TrendTracker module in Bluehill Central allows you to quickly search, display, and analyze results over time and across multiple samples and test systems. This document is intended to outline the statistics and charts available in TrendTracker.
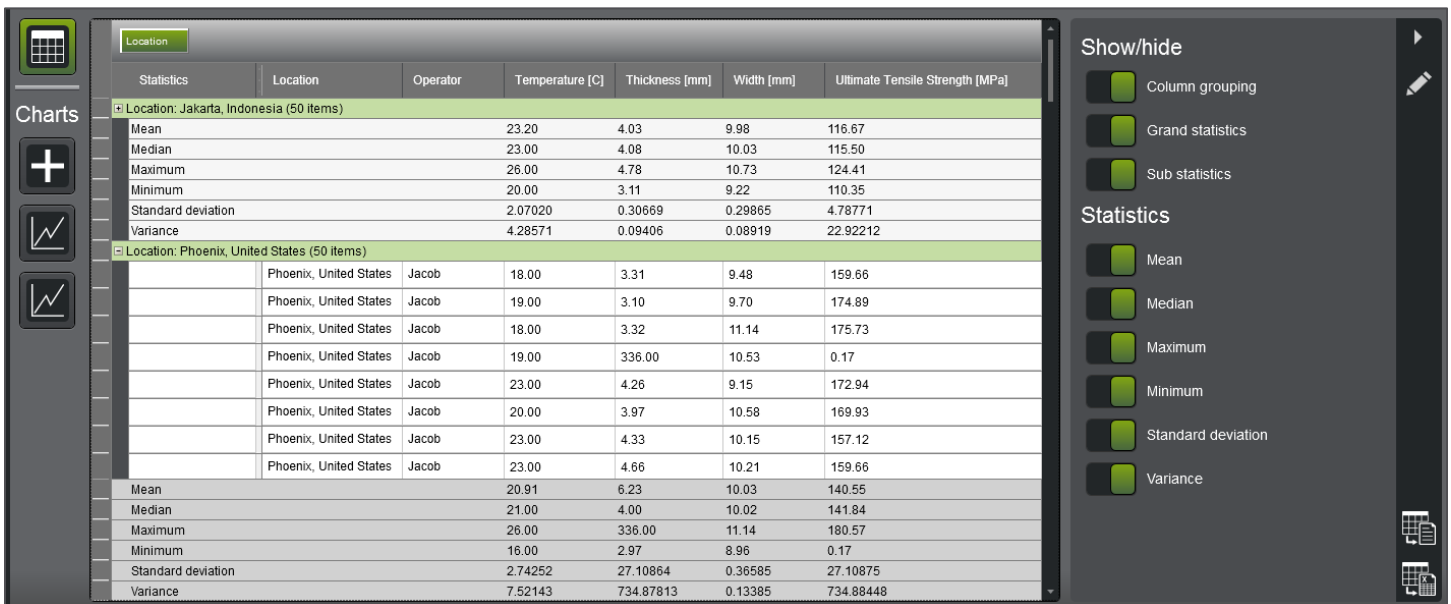
TABLE OF SPECIMEN RECORDS

All specimen records that meet the defined search criteria will be displayed in the table.
There are six available statistics that can be shown or hidden:

- Mean
- Median
- Maximum
- Minimum
- Standard deviation
- Variance

Table icon

Grand statistics and sub-statistics can be shown or hidden. Grand statistics are associated to all records while sub-statistics are associated to each grouping, if groups are created. To creating groupings to view sub-statistics, drag a column header to the column grouping header at the top of the table. In the screenshot below, the records have been grouped by "Location". Groups can be nested by dragging two or more column headers to the grouping header.

Ex. table of records

AVAILABLE CHARTS AND STATISTICS

TrendTracker offers several chart types to help visualize the specimen records and associated statistics as outlined below:

Chart type	Description	Sub-types	Available statistical tools

Individual chart	Individual charts are used to plot data associated to a specified metric, such as modulus, yield stress, break force, or width.	• Run chart
		• Histogram
			• Mean
			• Upper and lower acceptance limits
			• Upper control limit (UCL)
			• Lower control limit (LCL)
			• Standard deviation lines
			  o Mean ±1 STD
			  o Mean ±2 STD
			  o Mean ±3 STD
			• Moving range chart
			• Process Capability Metrics
			  o Pp
			  o Ppk
			• Rule check
			  o None
			  o WECO rules
			  o Basic rules

Group chart	Group charts are used to plot data associated to a specified metric and grouping. Groups are unitless parameters, such as system ID, batch #, sample name, or user.	• Xbar chart
		• R chart
		• S chart
		• Box plot
		• Histogram
			• Mean
			• Upper and lower acceptance limits
			• Process Capability Metrics
			  o Pp
			  o Ppk
			  o Cp
			  o Cpk
			• Upper control limit (UCL)
			• Lower control limit (LCL)
			• Rule check
			  o None
			  o WECO rules
			  o Basic rules

'Add new chart' and 'Select existing chart' icons

STATISTICAL DEFINITIONS

Population

As Instron defines

A population is the number of statistical units sharing at least one common property which is of interest in a statistical analysis.

Population 1

Sample

As Instron defines

A group of material specimens, whose properties are studied and compared to gain statistical or quality assurance information. For example, you could take a specimen from different parts of a single manufacturing run of a material to form a sample of the material. The sample is then representative of the complete run and you can analyze it to ensure that the material quality has remained stable over the complete run. You perform the same test on each specimen in a sample.
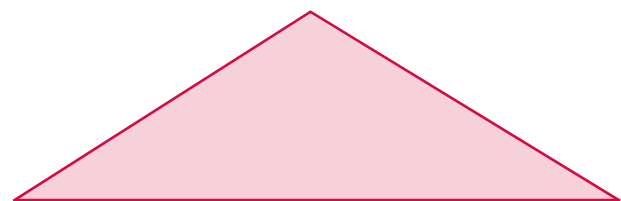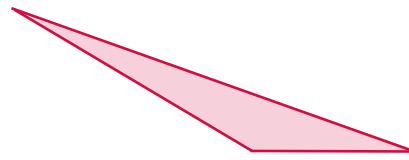
Sample 1   Sample 2   Sample 3   Sample 4   Sample 5

Specimen

As Instron defines

A single piece of a material for testing.

Specimen 1   Specimen 2   Specimen 3   Specimen 4   Specimen 5

Population Mean (μ)

The average of a result from all the specimens in a population.

$$Mean = \mu = \frac{\sum x_i}{n}$$

Where,

- μ is the sample mean
- x_i is the ith value
- $\sum x_i$ is summation of x_i
- n is the number of specimens in the population

Sample Mean (X bar, $\bar{X}$)

The average of a result from all the specimens in a sample.

$$X\ bar = \bar{X} = \frac{\sum X_i}{n}$$

Where,

- $\bar{X}$ is the sample mean
- X_i is the ith value
- $\sum X_i$ is the summation of X_i
- n is the number of specimens in the sample

Using force at break in Sample 1 as an example:
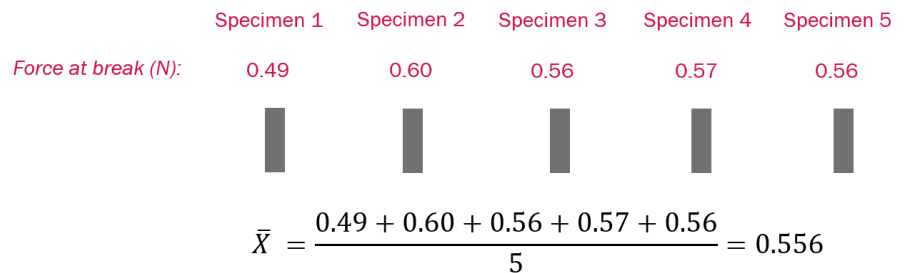
	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5
Force at break (N):	0.49	0.60	0.56	0.57	0.56

$$\bar{X} = \frac{0.49 + 0.60 + 0.56 + 0.57 + 0.56}{5} = 0.556$$

Population Standard Deviation (σ)

The spread of a result from its mean value from all the specimens within a population.

$$Standard\ deviation = \sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$$

Where,

- σ = standard deviation of the population
- μ = mean value of the population
- x_i = value of the ith specimen
- n = total number of specimens

Sample Standard Deviation (σ_i)

The spread of a result from its mean value from all the specimens within a sample.

$$Standard\ deviation = \sigma_i = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{X}_i)^2}{n-1}}$$

Where,

- σ_i = standard deviation of the ith sample group
- $\bar{X}_i$= mean value of the ith sample group
- x_i = value of the ith specimen
- n = total number of specimens

Using force at break in Sample 1 as an example:

Sample 1

$Mean = \bar{X}_1 = 0.556$

$$\sigma_1 = \sqrt{\frac{(0.49-0.556)^2 + (0.60-0.556)^2 + (0.57-0.556)^2 + (0.56-0.556)^2 + (0.57-0.556)^2}{5-1}}$$

$\sigma_1 = 0.0408$

Population Variance

The spread of a result from its mean value from all the specimens within a population.

$$Population\ variance = \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{X})^2}{n}$$

Where,

- σ^2 = variance of the population
- $\bar{X}$ = mean value of the population
- x_i = value of the ith specimen
- n = total number of specimens

Sample Variance

The spread of a result from its mean value from all the specimens within a sample.

$$Sample\ variance = \sigma_i^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{X}_i)^2}{n-1}$$

Where,

- σ_i = standard deviation of the ith sample group
- $\bar{X}_i$= mean value of the ith sample group
- x_i = value of the ith specimen
- n = total number of specimens

Sample Range (R)
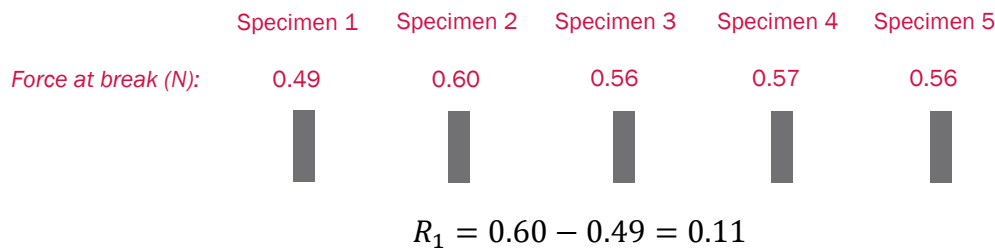
The difference between the maximum and minimum values of a result within a sample.

$$Range = R_i = X_{max_i} - X_{min_i}$$

Where,

- R_i = range value of the ith sample group
- X_{max_i} = maximum value in the ith sample group
- X_{min_i} = minimum value in the ith sample group

Using force at break in Sample 1 as an example:

	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5
Force at break (N):	0.49	0.60	0.56	0.57	0.56

$$R_1 = 0.60 - 0.49 = 0.11$$

Upper and Lower Control Limits

Control limits are horizontal lines displayed on control charts to represent process variation to help indicated when your process is out of control. Specification, or acceptance, limits are different from control limits. Control limits are based on actual product performance while specification limits are predetermined customer or internal requirements. A process can be in control and yet not be capable of meeting specifications.

Individuals Control Limits

$$Lower\ Control\ Limit, LCL = X - 2.66\ \overline{MR}$$

$$Upper\ Control\ Limit, UCL = X + 2.66\ \overline{MR}$$

Where X is the average of all data points and $\overline{MR}$ is the average of all the moving ranges.

Moving Range Control Limits

$$Lower\ Control\ Limit, LCL = 0$$

$$Upper\ Control\ Limit, UCL = 3.267\ \overline{MR}$$

Where $\overline{MR}$ is the average of all the moving ranges.

Xbar chart with fixed-size grouping

$$Lower\ Control\ Limit, LCL = \bar{X} - A_2\bar{R}$$

$$Upper\ Control\ Limit, UCL = \bar{X} + A_2\bar{R}$$

Where $\bar{X}$ is the average of all fixed-size averages, $\bar{R}$ is the average of all fixed-size ranges, and A_2 is a fixed-size-dependent factor found in the following table.

R chart with fixed-size grouping

$$Lower\ Control\ Limit, LCL = \bar{R}D_3$$

$$Upper\ Control\ Limit, UCL = \bar{R}D_4$$

Where $\bar{R}$ is the average of all fixed-size ranges and D_3 and D_4 are fixed-size-dependent factors found in the following table.

S chart with fixed-size grouping

$$Lower\ Control\ Limit, LCL = \bar{s} - 3\frac{\bar{s}}{c_4}\sqrt{1 - c_4^2}$$

$$Upper\ Control\ Limit, UCL = \bar{s} + 3\frac{\bar{s}}{c_4}\sqrt{1 - c_4^2}$$

Where $\bar{s}$ is the average of all fixed-size standard deviations and c_4 is a fixed-size-dependent factor found in the following table.

Fixed size, n	A_2	D_3	D_4	c_4
2	1.88	0	3.27	0.80
3	1.02	0	2.57	0.89
4	0.73	0	2.28	0.92
5	0.58	0	2.11	0.94
6	0.48	0	2.00	0.95
7	0.42	0.08	1.92	0.96
8	0.37	0.14	1.86	0.96
9	0.34	0.18	1.82	0.97
10	0.31	0.22	1.78	0.97
11	0.29	0.26	1.74	0.98
12	0.27	0.28	1.72	0.98
13	0.25	0.31	1.69	0.98
14	0.24	0.33	1.67	0.98
15	0.22	0.35	1.65	0.98
16	0.21	0.36	1.64	0.98
17	0.20	0.38	1.62	0.98
18	0.19	0.39	1.61	0.99
19	0.19	0.40	1.60	0.99
20	0.18	0.41	1.59	0.99
21	0.17	0.42	1.57	0.99
22	0.17	0.43	1.57	0.99
23	0.16	0.44	1.56	0.99
24	0.16	0.45	1.55	0.99
25	0.15	0.46	1.54	0.99

Factors for calculating limits for Xbar, R, and S charts

Process Capability and Performance Metrics

The process capability metrics are provided to support quality improvement methodologies such as Six Sigma, Design for Six Sigma (DFSS), and Lean Principles. They appear in forecast charts when a forecast definition includes a lower specification limit (LSL), upper specification limit (USL), or both[1]. TrendTracker refers to LSL and USL as lower and upper *acceptance* limits.

The main difference between P_p vs. C_p and P_{pk} .vs. C_{pk} is whether you use complete set of data, or population, for calculation (P_p and P_{pk}) where you calculate real performance of the system, or you use a sample (pre-production, batch, logical subgroups) where you calculate capability of the process[2].

Capability Metrics - C_p and C_{pk}

C_p is the capability index, measuring how well the data fits between the specification limits (USL, LSL). It doesn't evaluate if the process is centered within the limits, only if it would fit if it was centered. C_{pk} is the centering capability index, measuring how well the data is centered between the specification limits (USL, LSL). It is recommended to use C_p and C_{pk} when you have a *sample*, not the population, and are testing the potential capability of a process to meet customer needs.[3]

$$C_p = \frac{|USL - LSL|}{6\sigma} \qquad C_{pk} = min\left(\frac{\bar{X} - LSL}{3\sigma}, \frac{USL - \bar{X}}{3\sigma}\right)$$

Where,

- σ = sample standard deviation

Performance Metrics - P_p and P_{pk}

P_p is the performance index, measuring how well data fits between the specification limits (USL, LSL). It doesn't evaluate if the process is centered within the limits, only if it would fit if it was centered. P_{pk} is the performance centering index, measuring how well the data is centered between the specification limits. Use P_p P_{pk} when you have the total population and are testing the performance of a system to meet customer needs.[4]

$$P_p = \frac{|USL - LSL|}{6\sigma} \qquad P_{pk} = min\left(\frac{\bar{X} - LSL}{3\sigma}, \frac{USL - \bar{X}}{3\sigma}\right)$$

Where,

- σ = population standard deviation

---

[1] https://docs.oracle.com/cd/E57185_01/CBREG/ch04s03.html
[2] https://sixsigmastudyguide.com/process-capability-pp-ppk-cp-cpk/
[3] https://www.statisticshowto.com/process-capability-analysis/
[4] https://www.statisticshowto.com/process-capability-analysis/

Western Electric Co (WECO) Rules

In process control, WECO rules are used for detecting out-of-control or non-random conditions on control charts.

The 8 rules listed below help distinguish unnatural from natural patterns:

| Rule #	Description (Detect shifts or patterns in process)
1	The point is outside one of the 3-sigma control limits.
2	Two of three points are outside and on the same side as one of the 2-sigma control limits.
3	Four of five points are outside and on the same side as one of the 1-sigma control limits.
4	Eight out of eight points are on the same side of the mean.
5	Six points in a row are increasing or decreasing.
6	Fifteen points in a row are within the 1-sigma control limits.
7	Fourteen points in a row are alternating direction.
8	Eight points in a row are outside either of the 1-sigma control limits

Basic Rules

The basic rule is only rule 1 from the WECO rules

Rule #	Description (Detect shifts or patterns in process)
1	The point is outside one of the 3-sigma control limits.

Control Charts

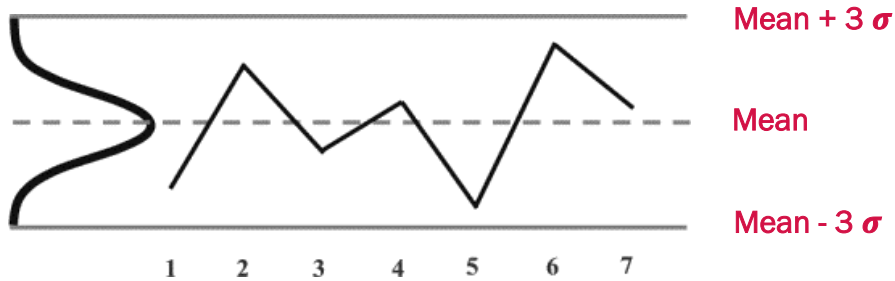Graphical and analytic tools for evaluating process variation over time.

Process Variability

Tendency of the measurement process to produce slightly different measurements on the same test specimen.

Source: http://www.infinityqs.com/resources/spc-tools
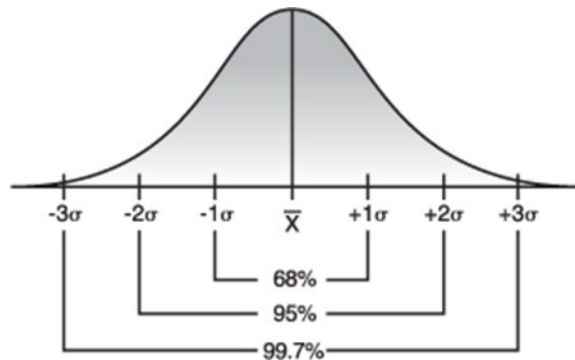
Rotate the distribution curve 90° to the right and extend the lines representing the mean and ± 3 standard deviations (σ). This forms the basis of Control Chart.

Mean + 3 σ

Mean

Mean - 3 σ

Source: https://www.moresteam.com/toolbox/statistical-process-control-spc.cfm

Individual Charts

Run Chart

Plots measurement for each specimen as a separate data point (i.e. sample size, n = 1).

Moving Range (MR) Chart

Plots the difference (range) between two consecutive data points to measure the process variability. One less data point compared to the I-chart. The moving range chart can be shown/hidden when using a run chart.
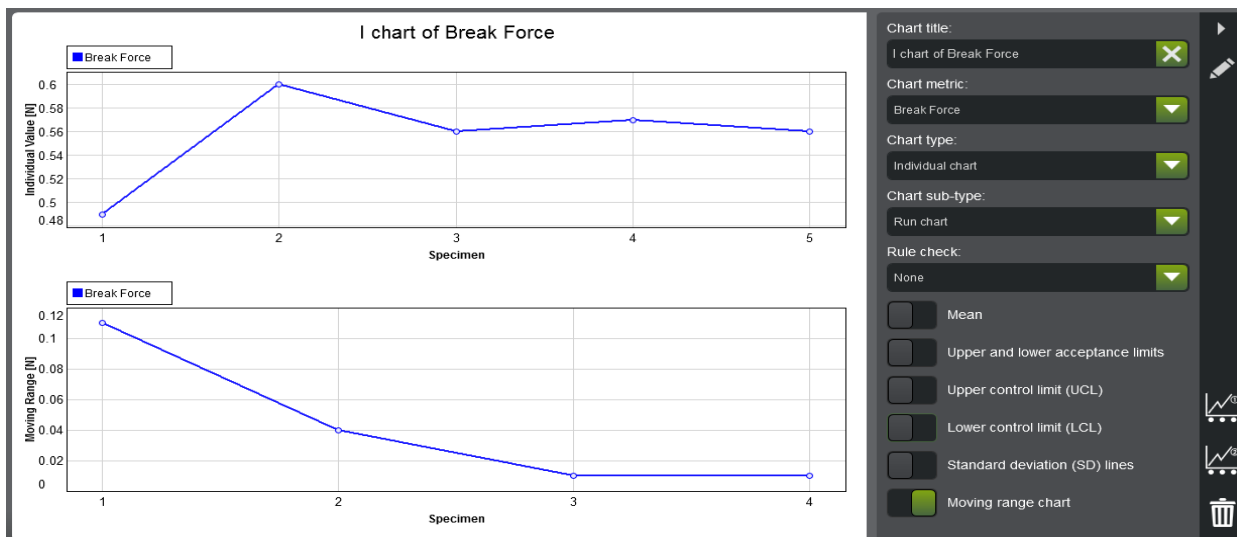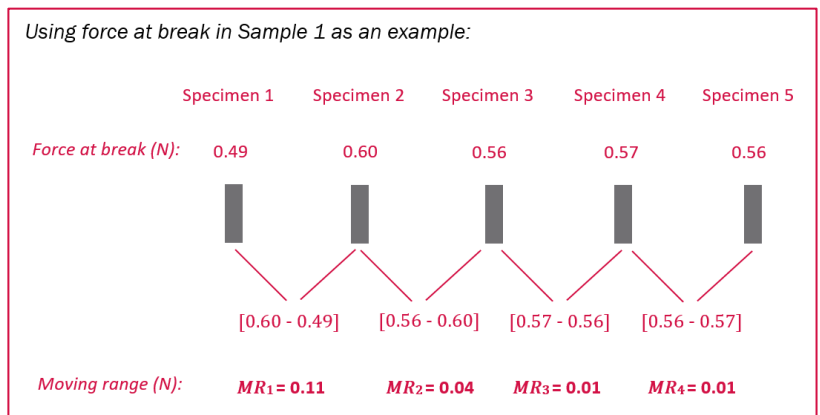
$$MR_i = |X_i - X_{i-1}|$$

Where,

MR_i = absolute difference between data point x_i & x_{i-1}
X_i = value of the ith measurement
X_{i-1} = value of the i^{th-1} measurement

Using force at break in Sample 1 as an example:

	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5
Force at break (N):	0.49	0.60	0.56	0.57	0.56

[0.60 - 0.49] [0.56 - 0.60] [0.57 - 0.56] [0.56 - 0.57]

Moving range (N): MR₁= 0.11 MR₂= 0.04 MR₃= 0.01 MR₄= 0.01

Example of Individual Run chart and Moving range charts

Histogram

Plots the frequency distribution of data by defined intervals, referred to as "bins" or "buckets".

$$Range = R_i = X_{max_i} - X_{min_i}$$

$$Width\ of\ bin = B_{width} = \frac{R_i}{n_B}$$

Where,

R_i = range value of the ith sample group

X_{max_i} = maximum value in the ith sample group

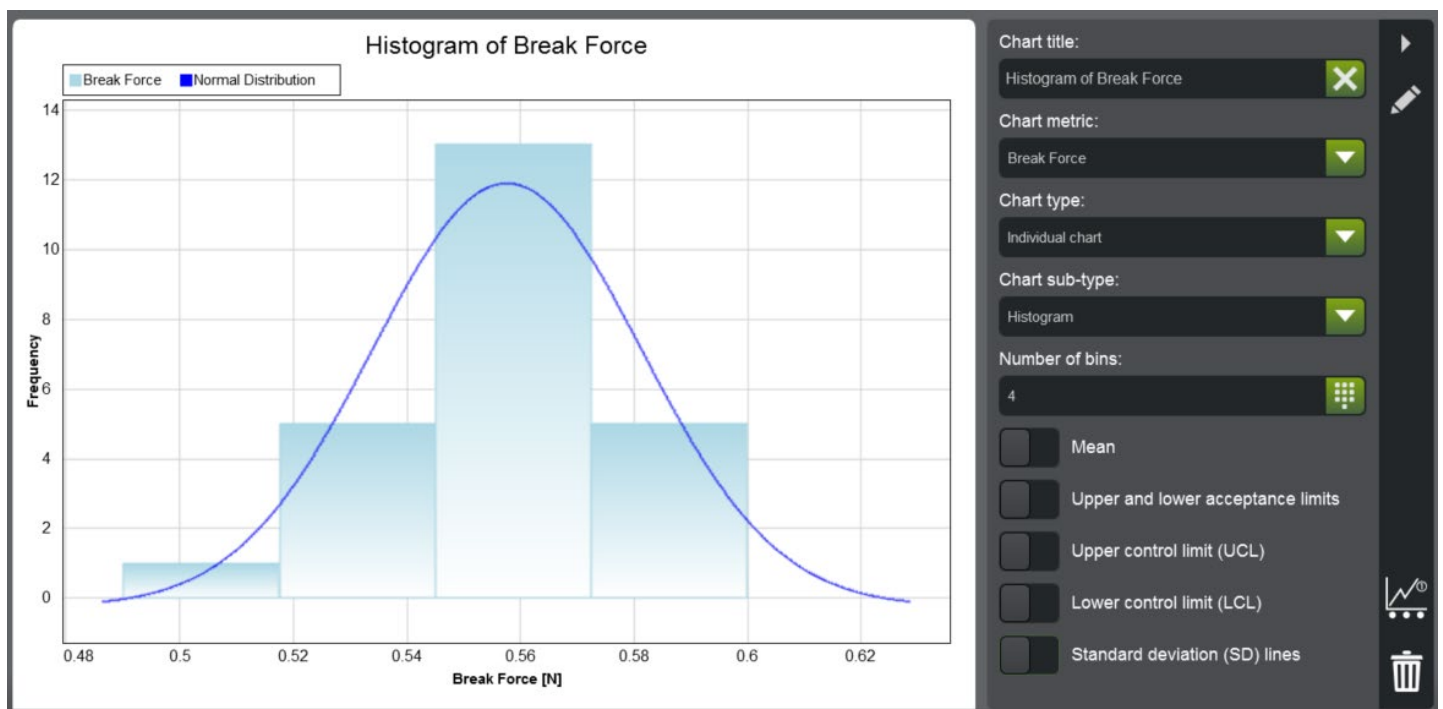X_{min_i} = minimum value in the ith sample group

n_B = number of bins specified

Using force at break in Samples 1 thru 5 as an example:

$$n_B = 4$$

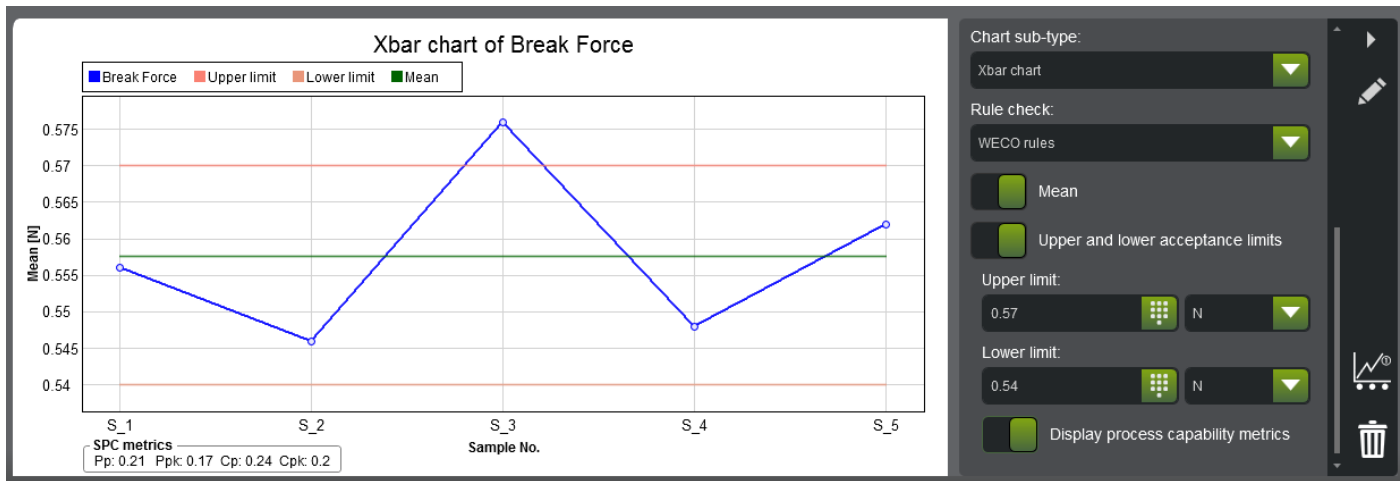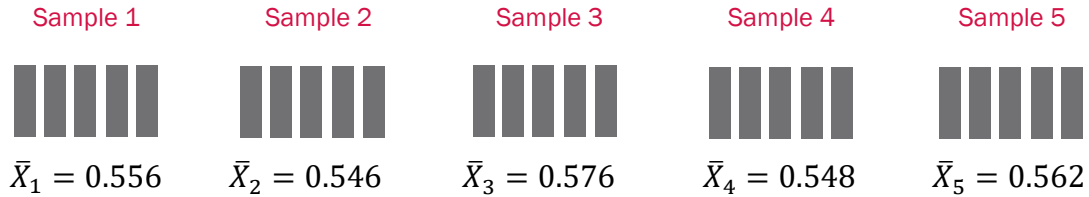$$R_i = 0.60 - 0.49 = 0.11$$

$$B_{width} = \frac{0.11}{4} = 0.0275$$

Example of individual histogram

GROUP CHARTS

X Bar Chart

Plot of the sample mean to evaluate changes in the mean of a process.

Using force at break in Samples 1 thru 5 as an example:

| Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
$\bar{X}_1 = 0.556$	$\bar{X}_2 = 0.546$	$\bar{X}_3 = 0.576$	$\bar{X}_4 = 0.548$	$\bar{X}_5 = 0.562$

Example of X Bar chart

R and S Charts

Both R (range) charts and S (standard deviation) charts analyze variability of a process. The R chart only uses the minimum and maximum values in each sample group while the S chart uses all the values in the group. The R chart is typically used when the sample size is relatively small (n < 10). For larger samples (n > 10), S chart is the preferred method to analyze data distribution.

R Chart

Plot of the sample range to evaluate variability of a process.
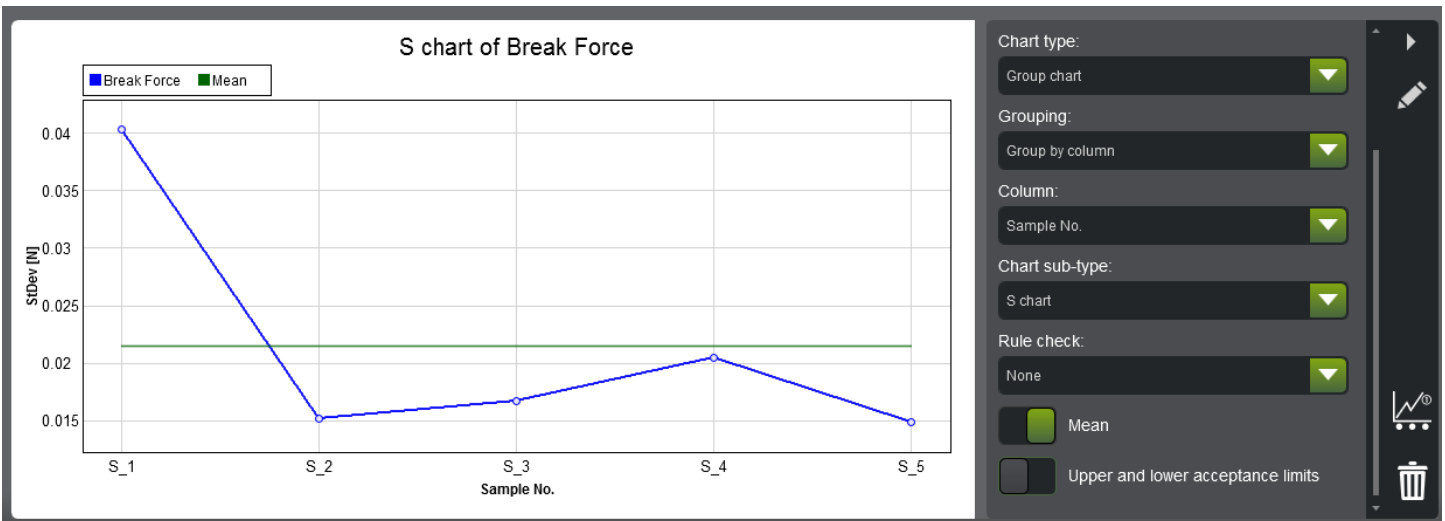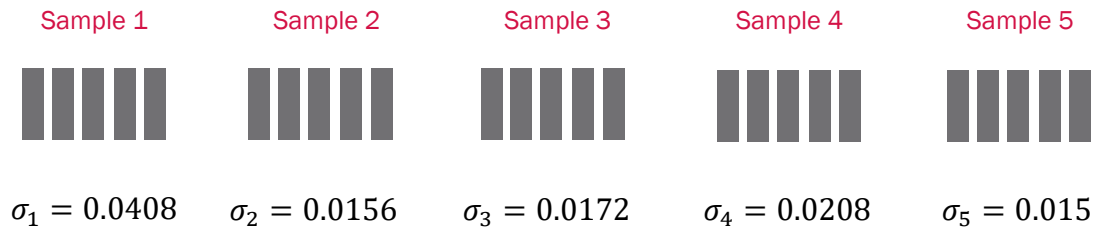
Using force at break in Samples 1 thru 5 as an example:

Sample 1	Sample 2	Sample 3	Sample 4	Sample 5
$R_1 = 0.11$	$R_2 = 0.03$	$R_3 = 0.04$	$R_4 = 0.05$	$R_5 = 0.04$
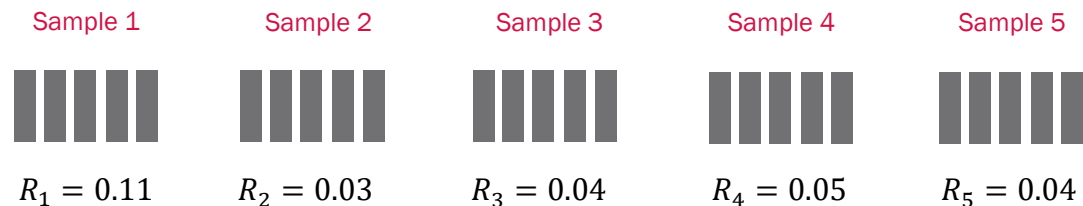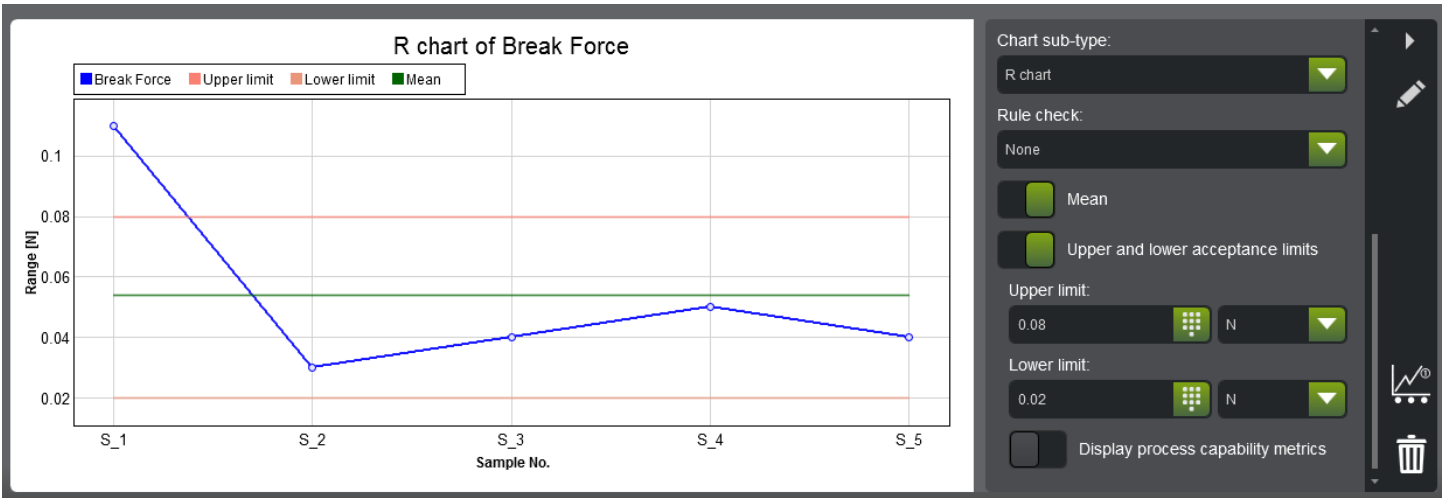
Example of R chart

S Chart

Plot of the sample standard deviation to evaluate variability of a process.

Using force at break in Samples 1 thru 5 as an example:

Sample 1	Sample 2	Sample 3	Sample 4	Sample 5

$$\sigma_1 = 0.0408 \quad \sigma_2 = 0.0156 \quad \sigma_3 = 0.0172 \quad \sigma_4 = 0.0208 \quad \sigma_5 = 0.015$$
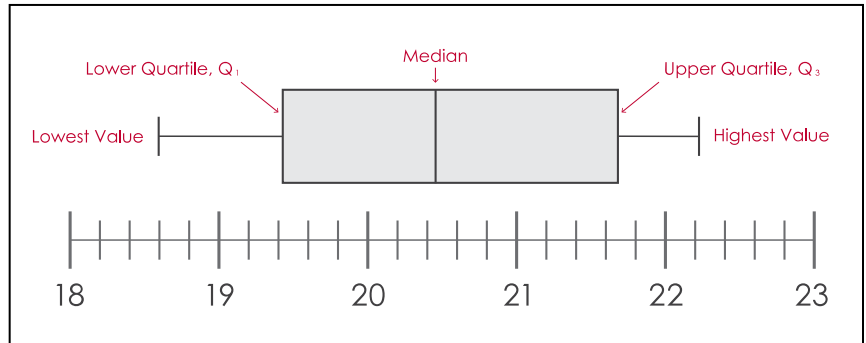

Example of S chart

Box Plot

Allows for comparison of several data sets simultaneously in the same graph. Also known as "box and whisker" plot. Box plots are constructed using 5 measurements:

1. *Minimum value*: lowest value in the data set
2. *Maximum value*: highest value in the data set
3. *Median*: middle value in the data set, divides the data into two halves
4. *Lower Quartile, Q1*: median of the lower half of the data set i.e. value below which the lower 25% of the data are contained
5. *Upper Quartile, Q3*: median of the upper half of the data set i.e. value above which the upper 25% of the data are contained


Using force at break in Sample 1 as an example:

	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5
Force at break (N):	0.49	0.60	0.56	0.57	0.56

1. Arrange the data in numerical order: 0.49, 0.56, 0.56, 0.57, 0.60
2. Min. value: 0.49, Max. value: 0.60, Median: 0.56
3. Find the lower and upper quartiles, Q1 & Q3.

Since "0.56" is already selected as the median, it is not included in the Q1 and Q3 calculations. The "box" goes from Q1 to Q3 value. "Whiskers" are drawn to the minimum and maximum values in the dataset. If min. value = Q1 (or max. value = Q3), the box plot will not have a bottom (or top) whisker.
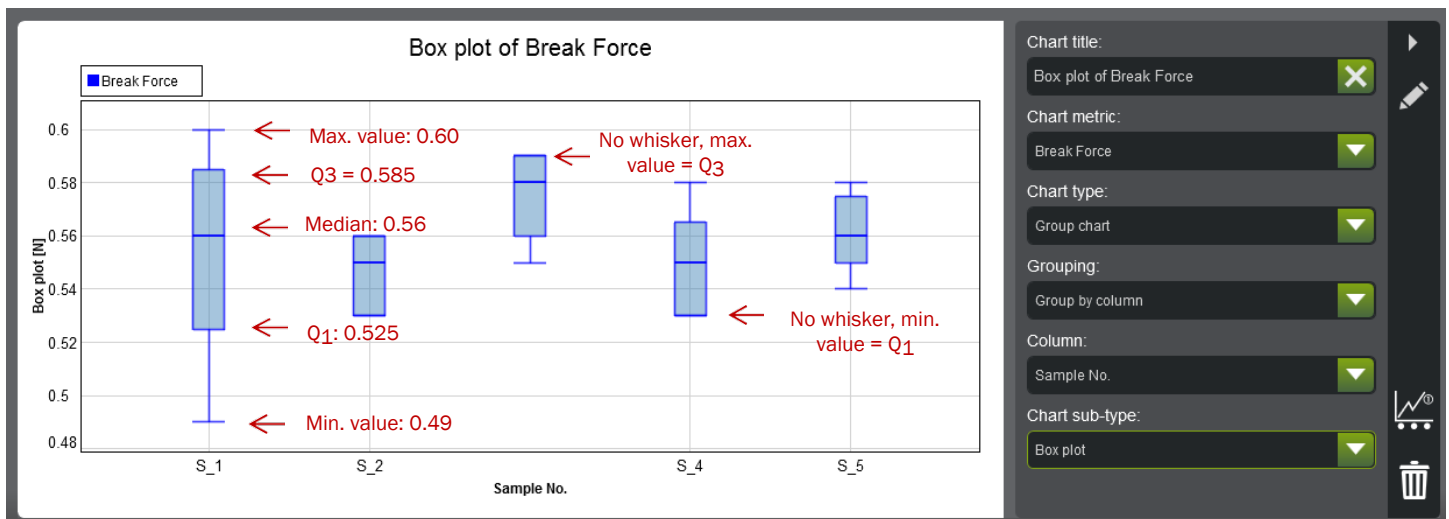
Example of Box plot with notations

Histogram

The histogram for group charts is similar to histograms for individual charts, the difference being the data in the histogram is the average ($\bar{X}_i$) of the group.

$$Range = R_{\bar{X}} = \bar{X}_{max_i} - \bar{X}_{min_i}$$

$$Width\ of\ bin = B_{width} = \frac{R_{\bar{X}}}{n_B}$$

Where,

$R_{\bar{X}}$= range of the groups' average values

$\bar{X}_{max_i}$= maximum average value in the ith sample group

$\bar{X}_{min_i}$= minimum average value in the ith sample group
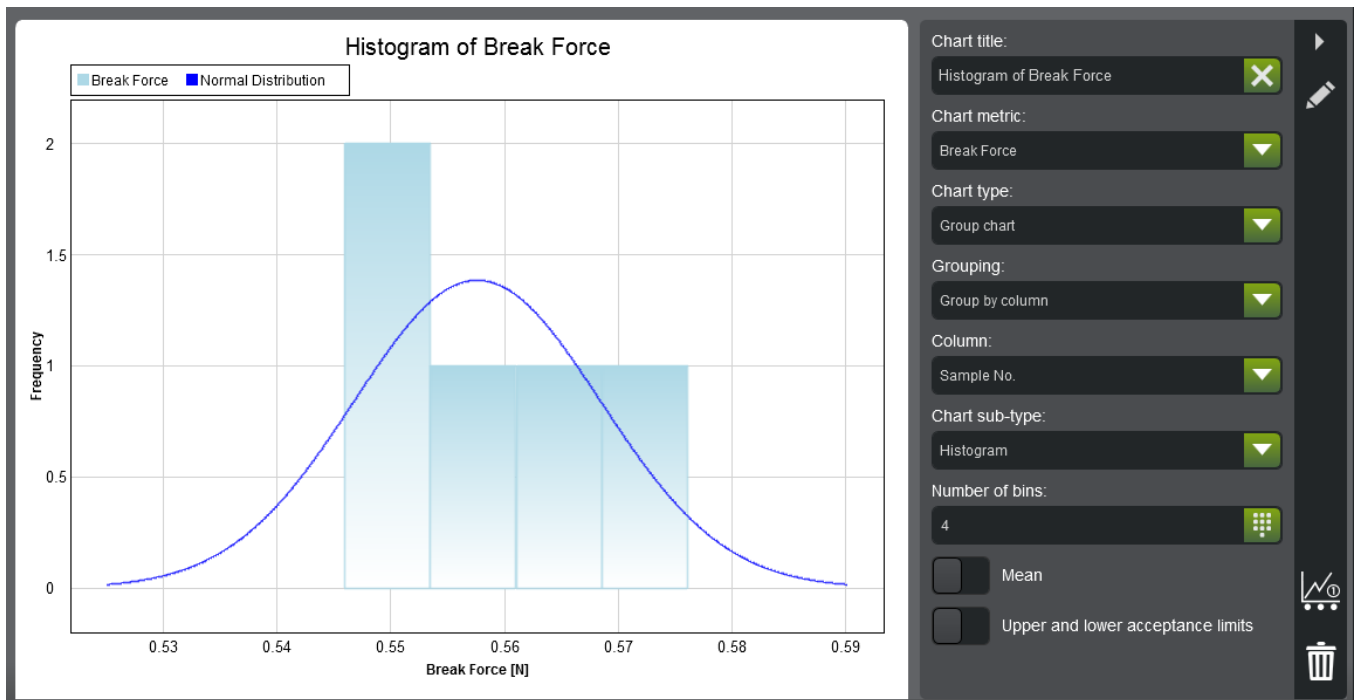
n_B= number of bins specified

Using force at break in Samples 1 thru 5 as an example:

$$n_B = 4$$

$$R_{\bar{X}} = 0.576 - 0.546 = 0.03$$

$$B_{width} = \frac{0.030}{4} = 0.0075$$

Example of group histogram